Fast alignment of a complex tracking detector using advanced track models

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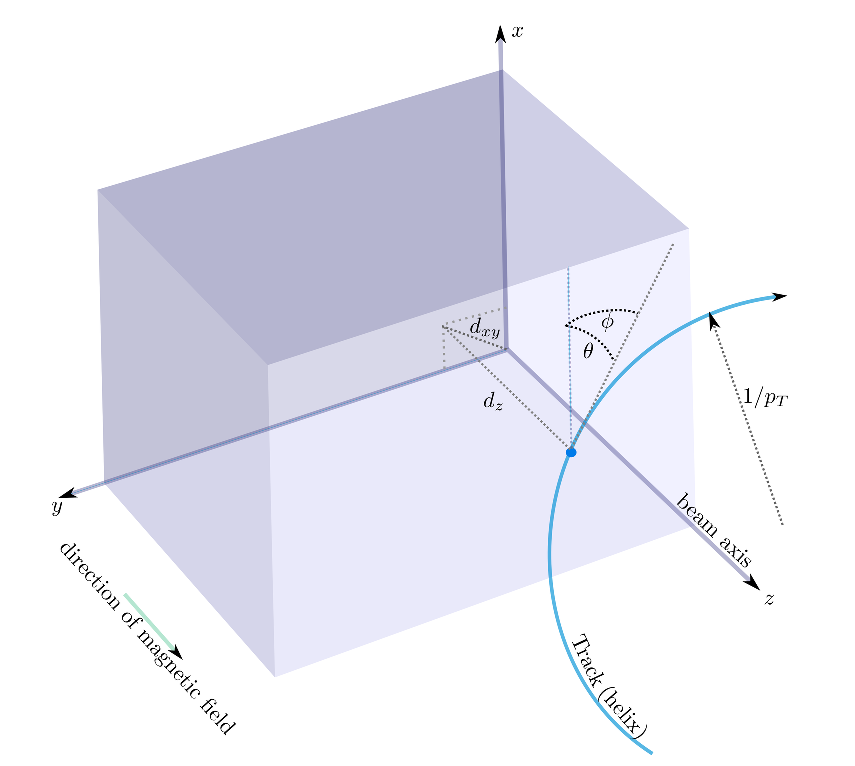
Abstract

The inner tracking silicon detector of the Compact Muon Solenoid experiment (CMS) at CERN's LHC collider consists of 16 588 modules. Charged-particle tracks are used to improve the accuracy of position and orientation of the modules. This contribution focuses on the Millepede-II algorithm, which is one of the two routinely used alignment algorithms in CMS [1]. Recently an advanced track model has been introduced in the CMS alignment based on "Broken Lines" and able to take the Multiple Coulomb Scattering in the detector material properly into account. We show the unique approach needed for solving the alignment problem in a reasonable amount of time on a routinely basis. Emphasis is given on the mathematical treatment of the problem.

The problem

A large number of particles is produced in the high-energy proton-proton collisions at the Large Hadron Collider (LHC). The inner tracker of the detector is designed to determine the track parameters of charged particles generated by the particle collisions. The track parameters for physics analysis are [a]



- the curvature $\kappa = q/p_T$ (expressed as signed inverse transverse momentum where q is the particle's charge)
- the impact parameter d_{xy} in the xy plane
- the impact parameter d_z along the principal axis of the experiment respectively
- the polar angles θ and ϕ.

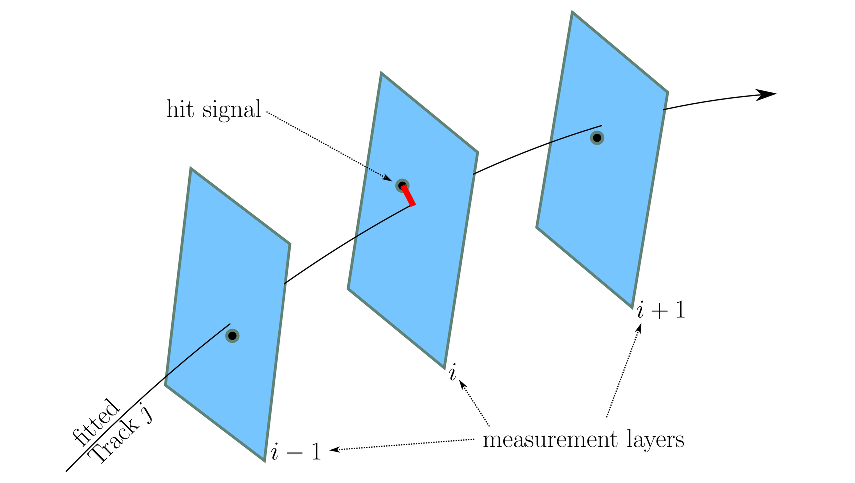
For each point along a track, a local coordinate system, the so-called *curvilinear frame*, is defined where the first coordinate is parallel to the track, the second one lies in the xy-plane and the third orthogonal to them, forming a right-handed coordinate system. Its origin along the track can be given by the arc length s.

For the accurate determination of the parameters of the tracks, the position of the modules forming the detector needs to be known to at least their intrinsic resolution, which is around $10\,\mu m$.

The algorithm should be reasonably fast, i.e. typical wall-clock time should be within a few hours.

[a] The CMS coordinate system is defined as follows[1]: The origin is at the nominal collision point, the x-axis pointing to the center of the LHC, the y-axis pointing up and the z-axis along the anticlockwise beam direction.

Track based alignment



Track based alignment can be described as a *least squares minimization* problem (χ^2 in high-energy physics parlance) where the data from hits generated by tracks are used. A single residual $\mathbf{r}_{ij}$ for hit i along track j is the three dimensional distance between the predicted hit location from the track model and the physical hit information from the modules, calculated using the actual knowledge of the geometry. Together with the covariance matrix $\mathbf{V}_{ij}$ the expression to be minimized is given in equation (1):

$$\chi^2(\mathbf{p},\mathbf{q}) = \sum_j^{\text{tracks}} \sum_i^{\text{hits}} \mathbf{r}_{ij}^T(\mathbf{p},\mathbf{q}_j)\,\mathbf{V}_{ij}^{-1}\,\mathbf{r}_{ij}(\mathbf{p},\mathbf{q}_j) \qquad (1)$$

where $\mathbf{p}$ denotes the alignment parameters describing the actual geometry and $\mathbf{q}_j$ denotes the track parameters of the j^{th} track. Due to the large number of alignment parameters and the required high alignment precision millions of tracks from different origins (collisions and "cosmics", i.e. muons produced in the outer atmosphere) have to be used. In addition survey information and other data like laser-alignment data can be added. The input data have to give stringent boundaries to the modules positions and they stabilize the solution of the problem.

Millepede-II

In Millepede the alignment parameters are determined in a simultaneous fit of all tracks, using a special method that reduces the size of the problem without the need to make approximations [2]. The χ^2 expression for the simultaneous fit for the determination of a large number of alignment parameters is given by the first-order Taylor expansion

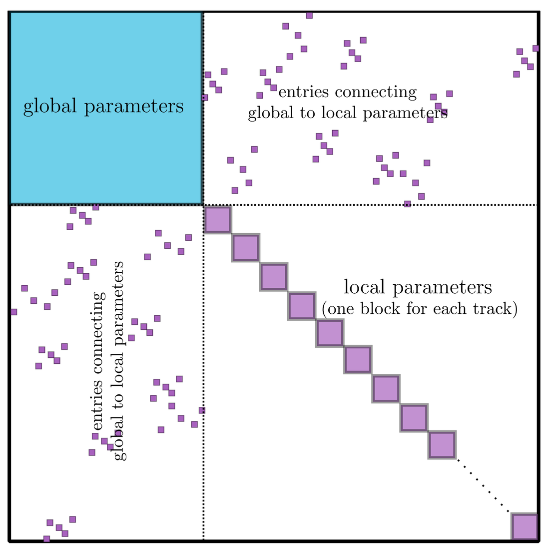
$$\chi^2(\mathbf{p},\mathbf{q}) = \sum_j^{\text{tracks}} \sum_i^{\text{hits}} \frac{1}{\sigma_{ij}^2}\left(\mathbf{m}_{ij} - \mathbf{f}_{ij}(\mathbf{p}_0,\mathbf{q}_{j0}) - \frac{\partial \mathbf{f}_{ij}}{\partial \mathbf{p}}\Delta\mathbf{p} - \frac{\partial \mathbf{f}_{ij}}{\partial \mathbf{q}_j}\Delta\mathbf{q}_i\right)^2 \qquad (2)$$

assuming uncorrelated measurements. The parameters involved split up into two groups:

Local parameters q: They describe the track used for the alignment. These may be the five track parameters mentioned above or another suitable parametrization, like the one presented below.

Global parameters p: They describe e.g. the position and orientation of the modules leading to six parameters, u, v, w for the position and α, β, γ as angles for the orientation.

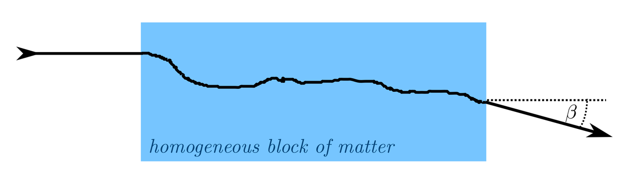
The local parameters of a single track are only connected to the subset of global parameters which are actually related to the particular track, and are not directly connected to the local parameters of other tracks. The matrix of the normal equations has therefore a special structure:



By applying block-matrix theorems, the huge matrix above can be rearranged, so that the problem is reduced to solving for the global parameters. The rearrangement requires the individual solution of all local fits; the inverse matrix of each local fit is necessary to update the global-parameter matrix according to the block-matrix theorems. Constraints from the physical structure of the problem are treated via Lagrange multipliers. The matrix equation for the global parameters, with a large sparse matrix, is solved by the fast iterative MINRES algorithm[3]. The solution is iterated for outlier rejection.

Broken lines

A charged particle traversing material experiences *multiple scattering*, mainly due to Coulomb interaction with the electrons in the atoms, resulting in a spatial shift and a change of the particle direction.

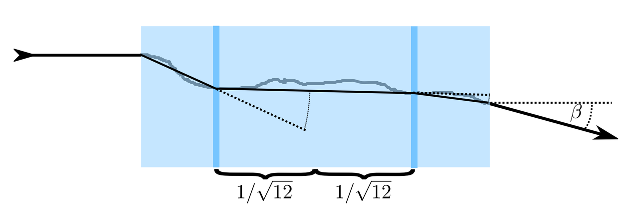


The mean of the deflection angle due to multiple scattering is $\langle\beta\rangle = 0$. The standard deviation $\sigma(\beta)$ can be approximated by the following formula[4]:

$$\sigma(\beta) = \frac{13.6\,\text{MeV}}{vp} z\sqrt{x/X_0}\,[1 + 0.038\ln(x/X_0)] \qquad (3)$$

where $v = \beta c$ (here β as the relativistic velocity factor) is the velocity of the particle, p its momentum and z the charge. x/X_0 is the thickness of the traversed medium in units of radiation lengths (the path length where the particle looses all but $1/e$ of its energy).

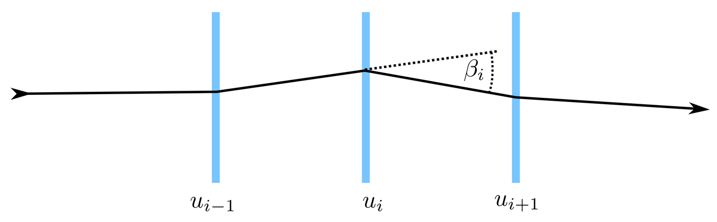
The effect depends not only on the amount of material, but also of the distribution of the material between the sensor planes. Thick scatterers are described by two equivalent thin scatterers (with same mean and RMS of the material distribution):



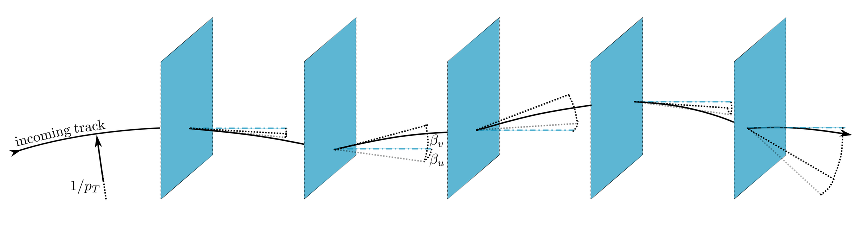
Broken lines *continued...*

In the special case of a silicon tracking detector, the material is concentrated at the sensor planes. Non-sensing material like cabling, cooling pipes, electronic circuitry etc. is regarded as part of the sensor.

A track propagating through the detector can now be described in the following way:



In the real three-dimensional case where every kink is described by two orthogonal angles along the propagation direction before the scatterer, this looks like



The propagation along the trajectory depends on the magnetic field. Also field inhomogeneities and the energy loss of the particle, which result in a change of the curvature κ along the track, have to be taken into account, see[5] for details. The mean value $\langle\beta_i\rangle$ will be the angle accumulated by the curvature while propagating. Particles are scattered away from its initial path in an omnidirectional way when viewed in its *curvilinear frame*, so the variance of the expectation values of the two angles to describe the scattering in three dimensions have the same value.

The expression to be minimized will be for n_{meas} hits and n_{scat} scatterers along one track

$$\chi^2(\kappa,\mathbf{u}) = \sum_{i=1}^{n_{\text{meas}}} (\mathbf{m}_i - \mathbf{P}_i\mathbf{u}_{\text{int},i})^T \mathbf{V}_{\text{meas},i}^{-1} (\mathbf{m}_i - \mathbf{P}_i\mathbf{u}_{\text{int},i}) + \sum_{i=2}^{n_{\text{scat}}-1} \boldsymbol{\beta}_i(\kappa,\mathbf{u})^T \mathbf{V}_{\beta,i}^{-1} \boldsymbol{\beta}_i(\kappa,\mathbf{u}) \qquad (4)$$

The vector of parameters $\mathbf{u} = (\mathbf{u}_1, \ldots \mathbf{u}_{n_{\text{scat}}})$ consists again of vectors of size 2. They describe the offset to the actual position in planes perpendicular to the track and are assumed to be reasonable small. Pixel and stereo strips provide two independent, single strip sensors only one measurement with a corresponding projection matrix $\mathbf{P}_i$ (of the offsets onto the measurement directions). The track is propagated by

$$\mathbf{u}_{\text{int},i} = \frac{s_n - s_i}{s_n - s_p}\mathbf{J}_{p,i}\mathbf{u}_p + \frac{s_i - s_p}{s_n - s_p}\mathbf{J}_{n,i}\mathbf{u}_n - \tfrac{1}{2}(s_n - s_i)(s_i - s_p)\mathbf{d}_{\text{mag}}\kappa \qquad (5)$$

where s_p and s_n are the previous/next neighbouring scatterers and $\mathbf{d}_{\text{mag}}$ is the deflection direction in the magnetic field, e.g. $(1, 0)$ for $\mathbf{B} = (0, 0, B_z)$ and $\mathbf{J}_{j,i}$ is the transformation from u_j to system of u_i.

The second sum consists of the deflection angles

$$\boldsymbol{\beta}_i = (\mathbf{J}_{i-1,i}\mathbf{u}_{i-1}\delta_{i-1} - \mathbf{u}_i(\delta_{i-1} + \delta_i) + \mathbf{J}_{i+1,i}\mathbf{u}_{i+1}\delta_i) - \tfrac{1}{2}(\Delta s_{i-1} + \Delta s_i)\mathbf{d}_{\text{mag}}\kappa \qquad (6)$$

where $\Delta s_i = s_{i+1} - s_i$, $\delta_i = 1/\Delta s_i$.

When setting up the normal equations for one track with local track parameters traversing n scatterers $\mathbf{q} = (\kappa, \mathbf{u}_1, \mathbf{u}_2, \ldots \mathbf{u}_n)$, the Jacobian matrix $\mathbf{A} = d\chi^2/d\mathbf{q}$ is calculated. The matrix $\mathbf{A}^T\mathbf{W}\mathbf{A}$ of the normal equations is a symmetric band matrix of band width $m = 5$, bordered by b full rows and columns.

The local fit is using a root-free Cholesky decomposition for the band part of the matrix with $O(n(m+b)^2)$ operations for the solution and $O(n^2(m+b))$ for the full covariance matrix (instead of a full inversion with $O(n^3)$). Curvature κ connects all u_i leading to a border of size $b = 1$. This procedure follows [6] and is implemented here for the first time in three dimensions. The special structure of the matrix allows also for the fast calculation of the full covariance matrix of all fit parameters, which is needed for the alignment.

The broken line method is faster than the reference algorithm for tracking, the Kálmán filter algorithm[7]. This is a sequential track fitting algorithm, which steps from scatterer to scatterer by adding measurements and so-called process noise (i.e. scattering), without calculating the full covariance matrix.

Acknowledgments

Image sources: CMS documents website (CMS drawings) and made by the authors (results and sketches).

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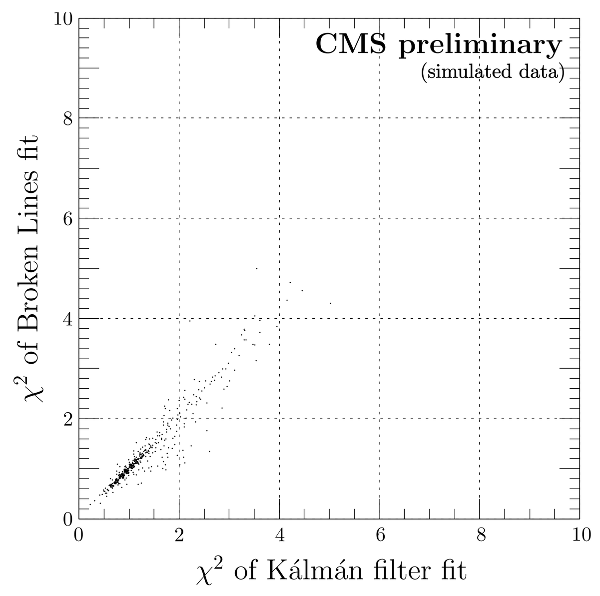
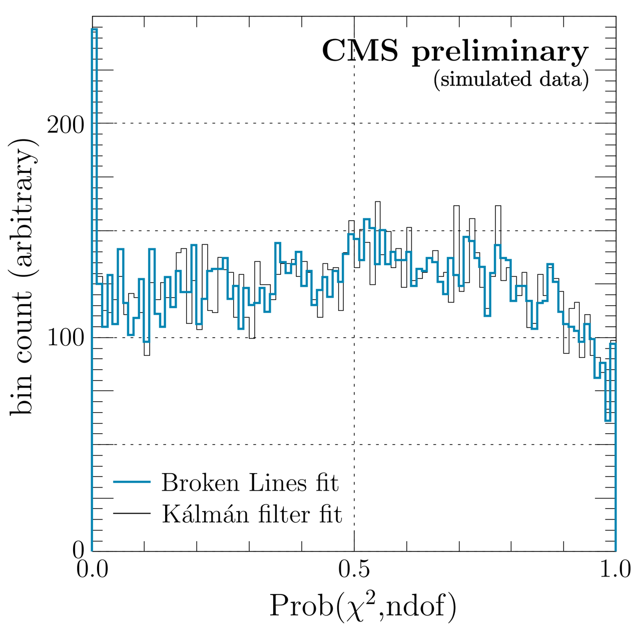
Results

Equivalence of track models. The results of the broken-line algorithm, used in the alignment procedure, and the standard Kálmán fit, used for track reconstruction in CMS, are equivalent. This shows a comparison of χ^2 values and P-values from track fits (obtained from 12 000 simulated tracks of type "isolated muon" using full detector reconstruction).



The probability of the χ^2 and the degrees of freedom ($ndof$) shows the same almost flat distribution for both track fit approaches:



Plotting these distributions versus track parameters (momentum, track angle) show no differences between the two fits (not shown here due to space restrictions).

Speed performance. Using a subset of the data (250 000 cosmic tracks), the performance of the solution by Cholesky decomposition was measured to be 7 times faster than full inversion of the matrix in the local fit.

Typical alignment of the full detector with about 4.5 millions of tracks and solving for 57 000 parameters takes 6 hours on one node (alignment algorithm only). Parallelization of some parts of the code using OpenMP™ improves speed on average by a factor of 3 (7 cores used). Typical memory consumption for such a job is up to 8 GB.

Conclusions

The use of a suitable track model for alignment has been shown. Several advantages have been demonstrated:

- Equivalence to the standard Kálmán filter fit approach
- Easily included into the existing Millepede-II algorithm
- A simple extension to the algorithm (Cholesky decomposition) allowed for performance optimization ($O(n(m+b)^2)$ instead of $O(n^3)$)
- Using parallelization, a speed improvement of a factor of 3 can be achieved.

This is already in routine use within CMS.

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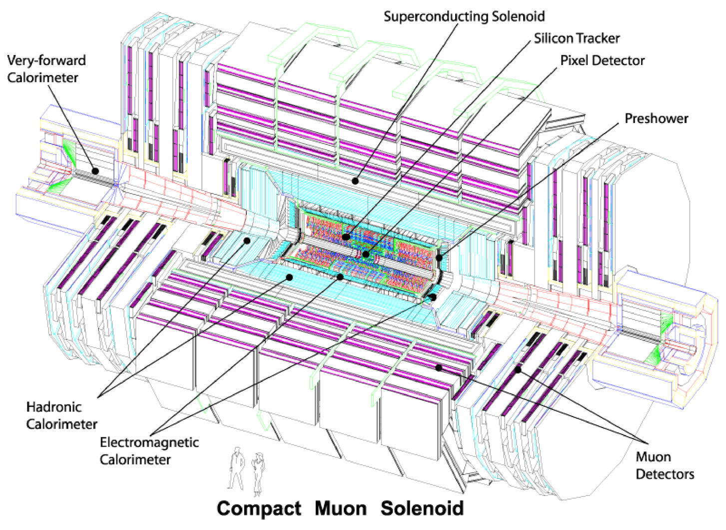
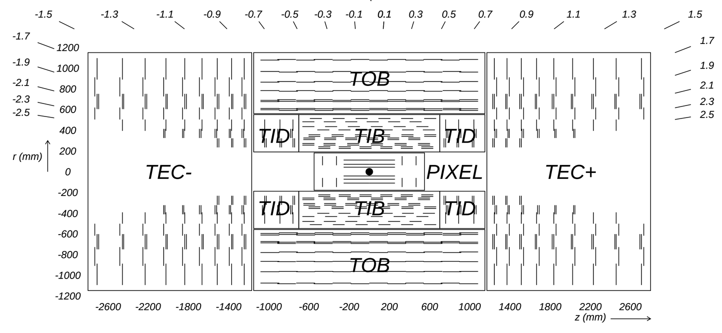
Millepede-II is maintained by the Helmholtz Alliance.
(https://www.wiki.terascale.de/index.php/Millepede_II)

The CMS experiment at the LHC and its inner tracker



The *Compact Muon Solenoid* (CMS) experiment is one of the *general purpose* experiments at CERN's *Large Hadron Collider* (LHC), located at the Swiss-French border near Geneva. It is designed to observe events generated from particle collisions to probe the *Standard Model of Particle Physics* and to search for new physics. Its main parts are an inner tracker made of silicon (pixels and strips), an electromagnetic and a hadronic calorimeter, a solenoidal coil to produce the magnetic field of about 3.8 T and an outer tracker to detect muons. The magnetic field is required to determine the charge and the momentum of the particles.



The inner tracker at CMS consists of 1440 silicon pixel modules and 15 148 silicon strip modules, grouped to sub-units as barrels and disks. Each module has six degrees of freedom (local coordinates u, v, w with respect to the geometric center of the module and rotations α, β, γ around these axes). In total we have to determine 69 232 parameters. For a typical alignment of the CMS inner tracker, around 10^6 to 10^7 tracks are required, depending on which hierarchy levels (modules or larger units) are selected as objects to be aligned. Therefore the number of parameters to be determined in this procedure becomes at least of the order $O(10^7)$. Two algorithms are in routine use, *Hits and Impact Points* and *Millepede-II*, both reducing the complexity to handle the problem on computers available to the experiment.

The sketch to the right shows how charged particles traverse CMS (transverse plane).